Losing Functions Without Gaining Data

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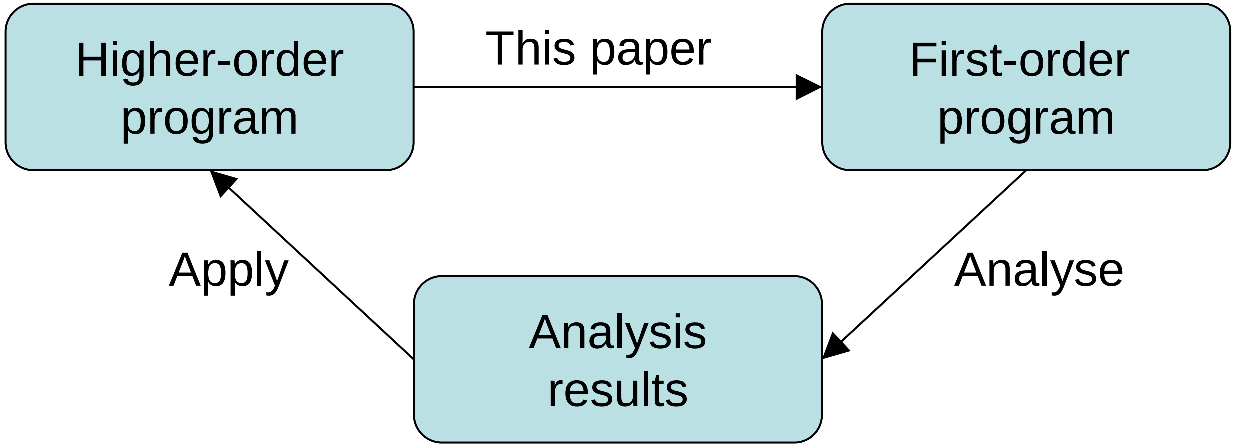
The Goal

- Remove functional values
  - Only named functions defined at the top level
  - No under/over application
- Without introducing data
  - Don’t want to introduce new data values
  - Avoid encoding functions in data

The Purpose

- Analysis!
  - Termination checking
  - Strictness analysis
  - Pattern-match safety (eg. Catch, Haskell08)



Example 1

```
sum :: [Int] → Int
sum xs = foldl (λx y → x + y) 0 xs

foldl :: (a → b → a) → a → [b] → a
foldl f z [] = z
foldl f z (x:xs) = foldl f (f z x) xs
```

Example 1: Result

$\text{sum} :: [\text{Int}] \rightarrow \text{Int}$

$\text{sum xs} = \text{foldl}_{+}\ 0\ \text{xs}$

$\text{foldl}_{+} :: a \rightarrow [b] \rightarrow a$

$\text{foldl}_{+}\ z\ []\ = z$

$\text{foldl}_{+}\ z\ (x{:}xs) = \text{foldl}_{+}\ (z + x)\ xs$

Ingredient: specialisation

Example 2

```
apply :: String → Int → Int
apply str x = case meaning str of
    Just f → f x
    Nothing → x

meaning :: String → Maybe (Int → Int)
meaning “abs” = Just abs
meaning _ = Nothing
```

Example 2: Result

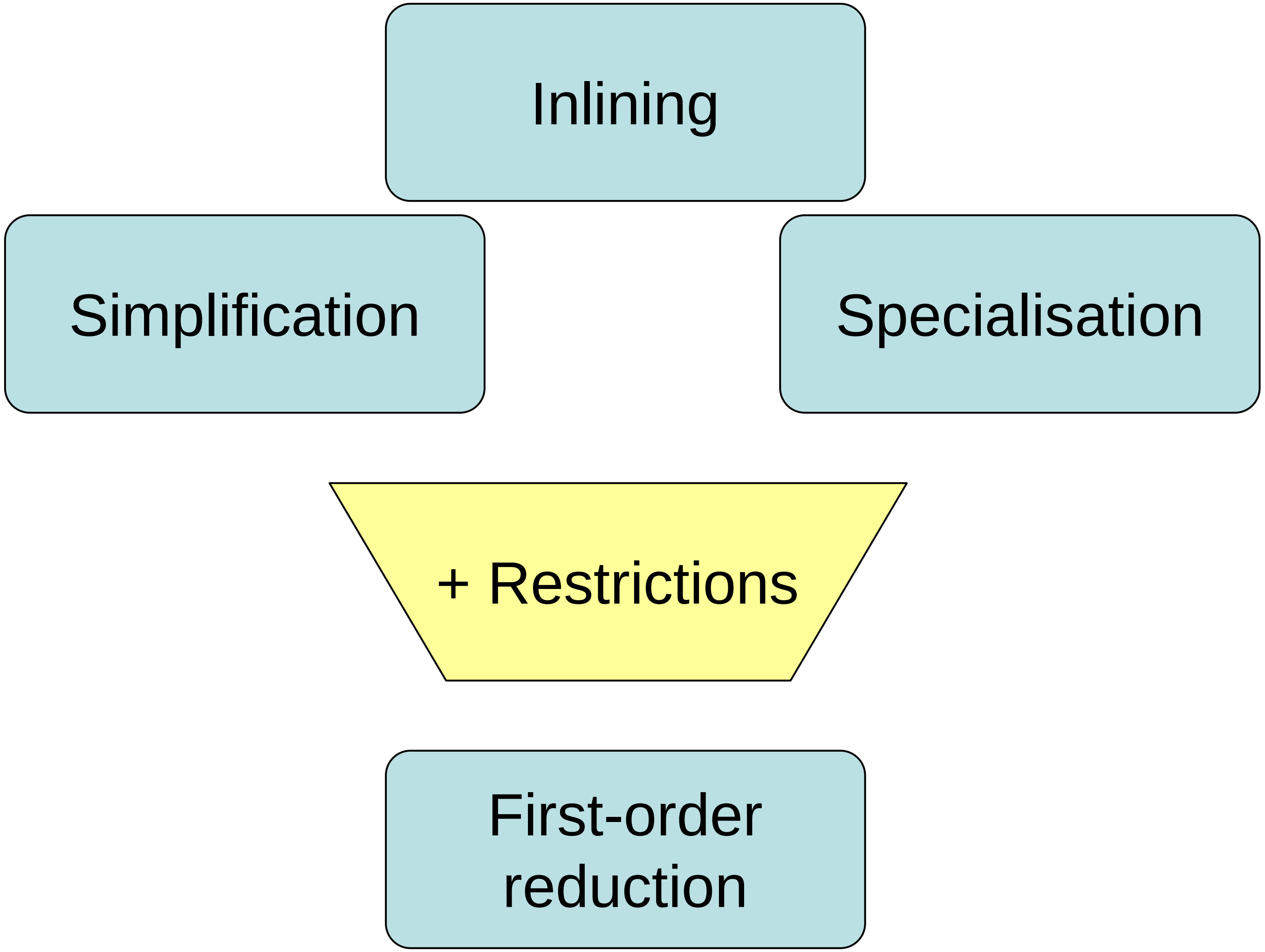
```
apply :: String → Int → Int
apply str x = case str of
    “abs” → abs x
    _ → x
```

Ingredients: inlining, simplification

The Central Idea

- Introduce explicit lambdas
  - Makes higher-order bits easier to see
- Move the lambdas around
  - The bulk of the work
- Eliminate lambdas
  - Applied lambdas
  - Unused lambdas

Moving Lambdas Around



Inlining

Simplification

Specialisation

+ Restrictions

First-order reduction

Purpose of Each Stage

- Simplification
  - Eliminate applied lambdas
- Inlining
  - Eliminate functions returning lambdas inside constructors
- Specialisation
  - Eliminate lambdas passed as arguments

Simplification

- Lots of basic simplifications
  - eg. case/case, case of known constructor, application of a lambda
- Also need these let rules
  - $\text{let } v = x \text{ in } \lambda w \rightarrow y \;\Rightarrow\; \lambda w \rightarrow \text{let } v = x \text{ in } y$
  - $\text{let } v = x \text{ in } y \;\Rightarrow\; y\,[x / v]$,
    if x is a lambda *or a boxed lambda*

Boxed Lambda

- Syntactic condition, under-approximates…
- …expressions whose results are constructions with a lambda component

Boxed Lambda's	Not Boxed Lambda's
[λx → x]	λx → [x]
Just [λx → x]	[foo (λx → x)]
let y = 1 in [λx → x]	foo [λx → x]
[Nothing, Just (λx → x)]	let v = [λx → x] in v

Inlining

- Purpose: eliminate functions returning boxed lambdas
- case f xs of … ⇒ case {body f} xs of …
  - where {body f} is boxed lambda

Specialisation

- Purpose: eliminate lambdas passed to functions
- Given f $e_1 \ldots e_n$, where some e_i is a lambda or boxed lambda
- Produce specialised f’
  - eliminate the i^{th} argument
  - introduce argument for each free variable in e_i
- Reformulate the application to use f’

Specialisation Example

1. sum xs = foldl (λx y → x + y) 0 xs
2. foldl_+ z xs = foldl (λx y → x + y) z xs
3. sum xs = foldl_+ 0 xs

Where the Lambdas Go

- Functions returning lambdas are eta expanded
- Functions returning boxed lambdas are inlined
- Functions with lambda arguments are specialised
- All other lambdas are targets for simplification rules

No lambda can hide!

Termination

- Specialisation may not terminate
  - Limited by homeomorphic embedding
- Inlining may not terminate
  - Limited by local numeric bounds

- Limits force termination when lambdas used to store an unbounded amount of information (eg. difference lists)

Disclaimers

- Not complete: may be residual lambdas if
  - Termination criteria kick in
  - Lambdas are passed to primitive functions
  - Root function takes/returns lambdas
- Loss of sharing

f x = let i = expensive x in λj → i + j

⇒

f x = λj → let i = expensive x in i + j

Results

- Successful on 62 of 66 nofib programs
  - Not cacheprof, grep, lift, prolog
- ~0.5 seconds to transform a program
  - Best = 0.1, Worst = 1.2
- Average code-size *reduction* of 30%
  - Best = 78% reduction, Worst = 27% increase
- Catch (Haskell08) relies on this method
  - 3 real bugs in HsColour

Results: Strictness

- Ask GHC – is add’s second arg strict?

```
add :: Int → Int → Int
add x y = apply 10 (+x) y

apply :: Int → (a → a) → a → a
apply 0 f x = x
apply n f x = apply (n - 1) f (f x)
```

Results: Termination

- Ask Agda – does this terminate?

```
cons : (N → List N) → N → List N
cons f x = x :: f x
downFrom : N → List N
downFrom = cons f
where f : N → List N
          f zero = [ ]
          f (suc x) = downFrom x
```

Conclusions

- Let’s analyse higher-order programs!

- Write first-order analysis pass
- Old way: extend to higher-order
  - ~5 years for strictness analysis
- New way: use defunctionalisation
  - ~0.5 seconds